Exercises for CIVIL-455 Transportation Economics: Traveler Choices under Uncertainty

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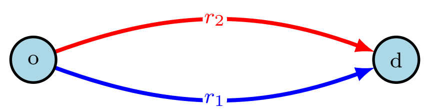
Q1 Consider a two-route network consisting of a route r_1 and a route r_2. The nature exists in one of two states: good and bad days. Tim has prior knowledge about the distributions of travel times on both routes:



	Good days	Bad days
Probability	80%	20%
Travel time on r_1	$10min$	$30min$
Travel time on r_2	$15min$	$15min$

(1) If Tim is risk-neutral and has no information about the actual state, which route would he choose?
(2) If Tim is risk-neutral and has perfect information about the actual state, which route would he choose? How much would Tim pay for perfect information?
(3) Suppose that the network manager provides traffic information to travelers with two signals: "congested" and "uncongested". When it is a good day, there is a 10% chance that the signal "congested" is sent. When it is a bad day, there is a 90% chance that the signal "congested" is sent. What is the unconditional probability of Tim being informed that it is congested?
(4) When Tim is informed that the network is "congested", what is his belief about the chance of today being a good day? Which route would he choose?
(5) How much would Tim pay for such information?
(6) Is any of these routes stochastically dominated by the other? What if the probability of good days changes?

Solutions:
(1) If Tim is risk-neutral and does not have information, he will choose route r_1 because the average travel time on route r_1 is

$$0.8 \times 10 + 0.2 \times 30 = 14 \leq 15.$$

(2) Under perfect information, Tim will choose route r_1 on good days and route r_2 on bad days. Therefore, the expected travel time for Tim would be

$$0.8 \times 10 + 0.2 \times 15 = 11.$$

Therefore, he is willing to pay $14 - 11 = 3$ for perfect information.
(3) The probability of sending signal "congested" is

$$\mathbb{P}(\text{congested}) = 0.8 \times 0.1 + 0.2 \times 0.9 = 0.26.$$

The probability of sending signal "uncongested" is

$$\mathbb{P}(\text{uncongested}) = 0.8 \times 0.9 + 0.2 \times 0.1 = 0.74.$$

Then, the unconditional probability of Tim being informed that it is congested is 26%.
(4) By Baye's rule, the belief of it being a good day upon receiving a signal "congested" is given by

$$\mathbb{P}(\text{good days} \mid \text{congested}) = \frac{0.8 \times 0.1}{0.8 \times 0.1 + 0.2 \times 0.9} = \frac{4}{13}.$$

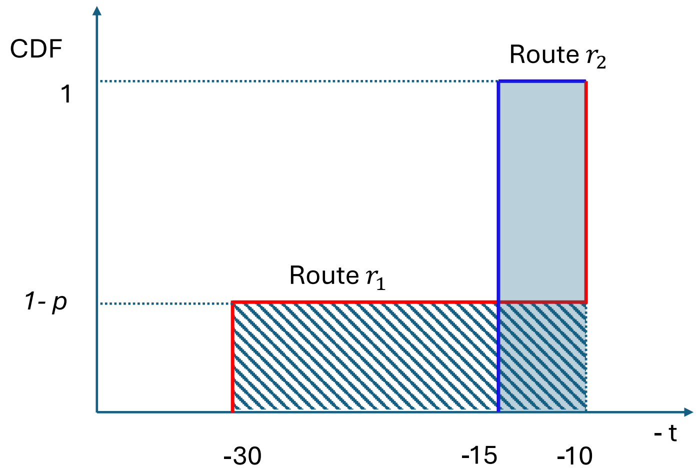
Upon receiving "congested", then the expected travel time of routes r_1 and r_2 are respectively $10 \times 4/13 + 30 \times 9/13 = 310/13$ and 15. Then Tim would choose route r_2 in this case. By Bayes' rule, the belief of it being a good day upon receiving a signal "uncongested" is given by

$$\mathbb{P}(\text{good days} \mid \text{uncongested}) = \frac{0.8 \times 0.9}{0.8 \times 0.9 + 0.2 \times 0.1} = \frac{36}{37}.$$

Upon receiving "uncongested", then the expected travel time of routes r_1 and r_2 are respectively $10 \times 36/37 + 30 \times 1/37$ and 15. Then Tim would choose route r_1 in this case.
(5) Thus, on good days, the expected cost of Tim is $0.9 \times 10 + 0.1 \times 15$. On bad days, the expected cost of Tim is $0.9 \times 15 + 0.1 \times 30$. Then the expected cost of Tim for all days is

$$0.8(0.9 \times 10 + 0.1 \times 15) + 0.2(0.9 \times 15 + 0.1 \times 30) = 11.7.$$

Then Tim can pay $14 - 11.7 = 2.3$ for such imperfect information.
(6) We can compare the areas under the blue curve and the red curve, respectively. For any $x \leq -15$, it is obvious that the area under the CDF of r_2 is lower than the area under r_1. When $x \geq -15$, the area under the CDF of r_2 increases faster than the area under the CDF of r_1. Thus, as long as the shadowed area ($20(1-p)$) is greater than the blue area (5), i.e., $p < 3/4$, the area under the CDF of r_2 is always lower than the area under r_1. By definition, r_1 is second-order stochastically dominated by r_2. That is, if the $p < 3/4$, route r_2 is preferred by risk-averse travelers.


CDF
Route r_2
1
Route r_1
1- p
- t
-30
-15
-10